DTU Library

Systematic equation formulation

Lindberg, Erik

Publication date:
2007

Document Version
Publisher's PDF, also known as Version of record

ECCTD 2007

Systematic Equation Formulation

Erik Lindberg, IEEE Lifemember

Ørsted•DTU Department, 348 Technical University of Denmark,
DK2800 Kongens Lyngby, DENMARK

E-mail: el@oersted.dtu.dk

Abstract

ECCTD 2007

A tutorial giving a very simple introduction to the set-up of the equations used as a model for an electrical / electronic circuit

The aim is to find a method which is as simple and general as possible with respect to implementation in a computer program

The "Modified Nodal Approach" MNA
and the "Controlled Source Approach" CSA
for systematic equation formulation are investigated

Abstract, continued

It is suggested that the kernel of the PSpice program based on MNA is reprogrammed

It is more than 30 years ago that the MNA was published

but

It is difficult to find a textbook which describe this equation formulation scheme

- INTRODUCTION
- THE “MODIFIED NODAL APPROACH” MNA
- THE “CONTROLLED SOURCE APPROACH” CSA
- AN EXAMPLE
- CONCLUSION

INTRODUCTION

analyze an electrical/electronic circuit

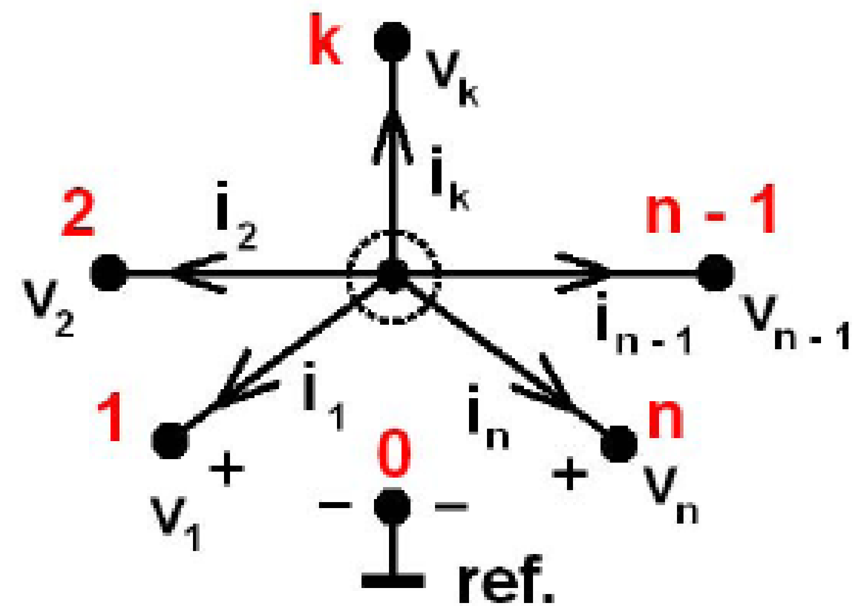
(1) formulate equations

(2) solution of the equations
in the time and frequency domains

(3) evaluation of the solution

The nodal formulation is based on the Kirchhoff Current Law, KCL: The sum of the currents leaving a closed subset of a circuit must be zero, e.g. the sum of the currents leaving an element (a branch, a two-terminal) is zero or the sum of the currents leaving a node is zero.

A closed subset of a circuit is a "super node" or a "multi terminal".



NODE EQUATIONS

KVL: $\underline{VB} = \underline{\underline{A}} * \underline{VN}$

KCL: $\underline{\underline{A}}^t * \underline{IB} = - \underline{JS}$

OHM: $\underline{\underline{YB}} * \underline{VB} = \underline{IB}$

$$\underline{\underline{A}}^t * \underline{\underline{YB}} * \underline{\underline{A}} * \underline{VN} = \underline{\underline{A}}^t * \underline{IB} = - \underline{JS}$$

$\underline{\underline{A}}$ = INCIDENCE MATRIX

$\underline{\underline{YB}}$ = BRANCH ADMITTANCE MATRIX

$\underline{\underline{YN}} = \underline{\underline{A}}^t * \underline{\underline{YB}} * \underline{\underline{A}}$ = NODE ADMITTANCE MATRIX

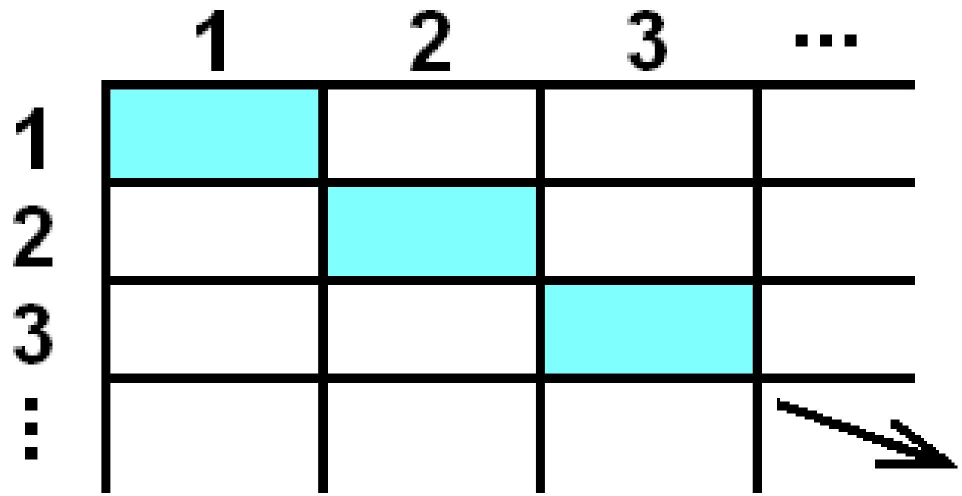
$$\underline{\underline{A}} = \begin{array}{c} N \longrightarrow \\ \begin{bmatrix} 0 & +1 & -1 \end{bmatrix} \end{array} \begin{array}{c} B \\ \downarrow \end{array} \qquad \underline{\underline{YB}} = \begin{bmatrix} \ddots & 0 \\ 0 & \ddots \end{bmatrix}$$

$$\boxed{\underline{\underline{YN}} * \underline{VN} = - \underline{JS}}$$

FORMULATION BY INSPECTION

- DIAGONAL ENTRIES ARE THE SUM OF ADMITTANCES INCIDENT TO NODE
- OFF-DIAGONAL ENTRIES ARE THE NEGATIVE SUM OF ADMITTANCES BETWEEN NODES
- SET-UP IN A SINGLE SCAN OF THE ELEMENT-LIST (BRANCH-LIST, NET-LIST)

"GROW" THE EQUATIONS



NODE EQUATIONS RESTRICTIONS DRAW BACKS

1. NO SHORT-CIRCUITS ($R = 0$, $G = \infty$)

2. NO IDEAL VOLTAGE SOURCES

3. NO IDEAL CURRENT-CONTROLLED SOURCES

4. INTEGRO-DIFFERENTIAL EQUATIONS

$\rightarrow$ 2. ORDER DIFFERENTIAL EQUATIONS

sC , G , $1/(sL)$

$$\underline{\underline{YN}} * \underline{VN} = - \underline{JS}$$

INTRODUCTION

PSpice problems

"Analog Behavioral Modeling" (ABM)

No equivalent "F" (CCCS) or "H" (CCVS) part types in the part library because PSpice "F" and "H" devices do not support the ABM extensions

The "E" (VCVS) and the "G" (VCCS) device types can not be controlled directly by the time derivative of the controlling signal (variable), instead a voltage source/capacitor implementation is used

PSpice do not implement the MNA in a systematic and simple way

Modified Nodal Approach

Ho, C.W., Ruehli, A.E., Brennan, P.A.

"The Modified Nodal Approach to Circuit Analysis ",

IEEE Transactions on Circuits and Systems,

Vol. CAS-22, June 1975, pp. 504-509.

Modified Nodal Approach

MNA

First the nodal equations are formulated

$$\underline{\underline{\mathbf{Y}}}\,\underline{\mathbf{V}} = \underline{\mathbf{J}}$$

where $\underline{\underline{\mathbf{Y}}}$ is the node admittance matrix, $\underline{\mathbf{V}}$ is the node potential vector (the common datum voltages) and $\underline{\mathbf{J}}$ the current source vector

Next the branch currents for voltage sources and other branches whose currents are controlling variables are introduced as additional variables

$$\begin{bmatrix} \mathbf{Y}_R & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{V}} \\ \underline{\mathbf{I}} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{J}} \\ \underline{\mathbf{F}} \end{bmatrix}$$

MNA

Y_R **is a reduced form of the nodal matrix excluding the contributions due to voltage sources, current controlling branches, etc.**

B **contains partial derivatives of the Kirchhoff current equations with respect to the additional current variables and thus contains ±1's for the elements whose branch relations are introduced**

The branch constitutive relations, differentiated with respect to the unknown vector, are represented by the matrices C **and** D

MNA

Tables with contribution "stamps" for network branches as e.g. coils L or capacitors C are given

For each of the branch types (element types) G , C and J there are two stamps depending on whether the branch current is an output variable or not

The branch current is always introduced as an additional variable for an inductor and a voltage source

For current sources J, resistors R, conductances G, and capacitors C, the branch current is only introduced as an additional variable under the following conditions:

1. if other nonlinear circuit elements depend on its current

and

2. if the branch current is requested as an output variable

Controlled Source Approach

How to overcome the

RESTRICTIONS DRAW BACKS

of the node equation formulation

$$\underline{\underline{YN}} * \underline{VN} = - \underline{JS}$$

Start from scratch

Controlled Source Approach

CSA

There are two kinds of electrical systems:

(1) Power systems for transport of energy and
(2) Electronic systems for transport of information

If you want to analyze these systems the first step is to set-up a model based on a choice of variables, i.e. you have to make assumptions concerning
the kind and number of variables
adequate for the analysis you want to make

The basic physical variables are charge q and flux φ which are difficult to measure, instead

current ($i = dq/dt$) and voltage ($v = d\varphi/dt$)
are used as basic variables

The Maxwell equations are the basic equations which can be used for modeling an electrical system

Two basic assumptions

the system is quasi-stationary

i.e. the size of the system is
small compared to the wavelength of the signals
so that a lumped element model could be used

the lumped elements of the system can be modeled as controlled sources

There are four kinds of controlled sources:

VCCS (Voltage Controlled Current Source)

VCVS (Voltage Controlled Voltage Source)

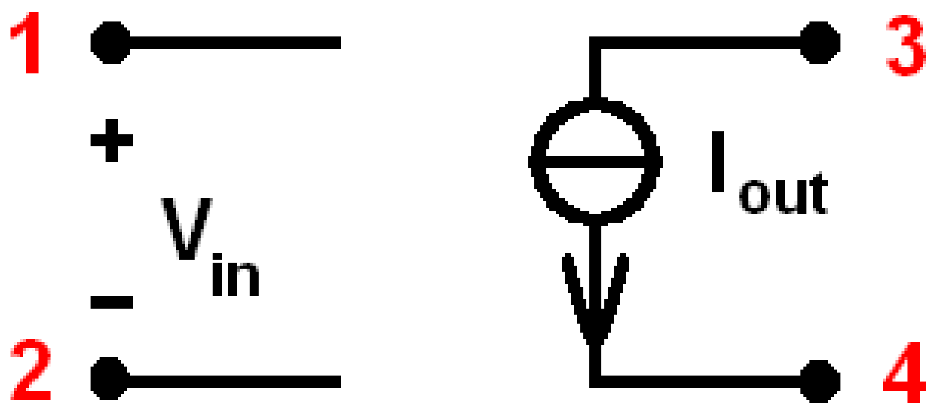
CCCS (Current Controlled Current Source)

CCVS (Current Controlled Voltage Source)

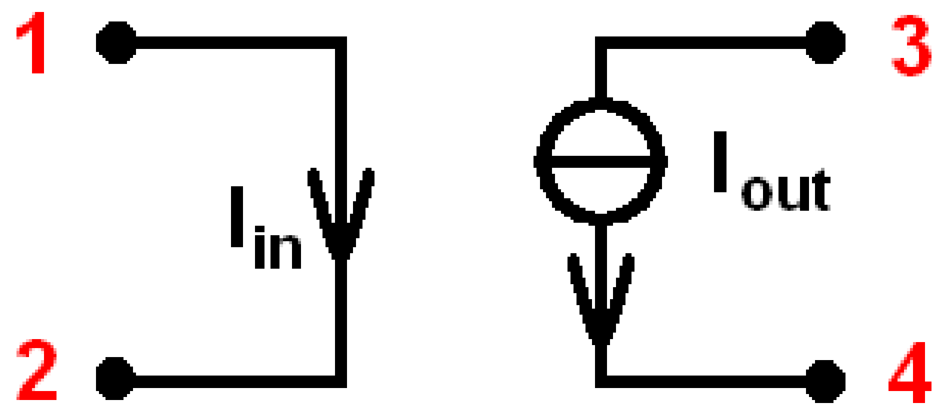
ni+ = 1, ni− = 2, no+ = 3, no− = 4

ni+ = 1, ni− = 2, no+ = 3, no− = 4

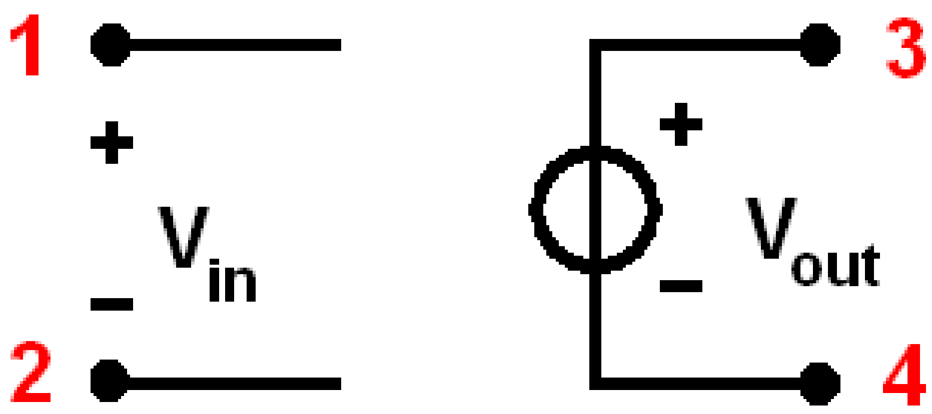
CSA



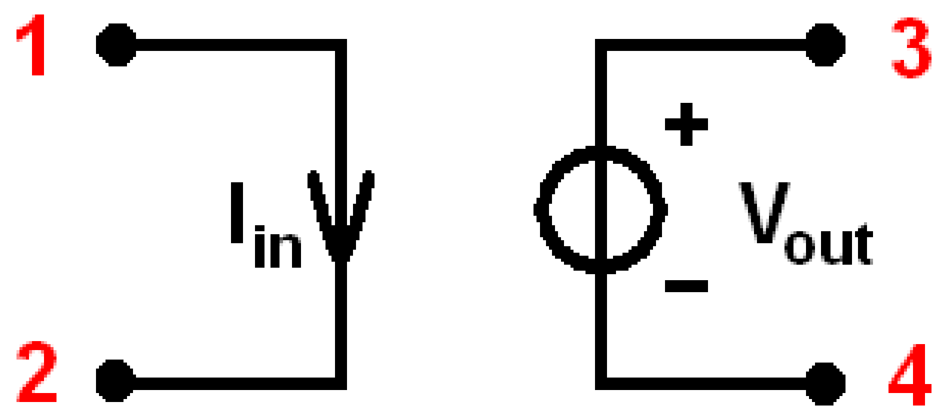
VCCS



CCCS



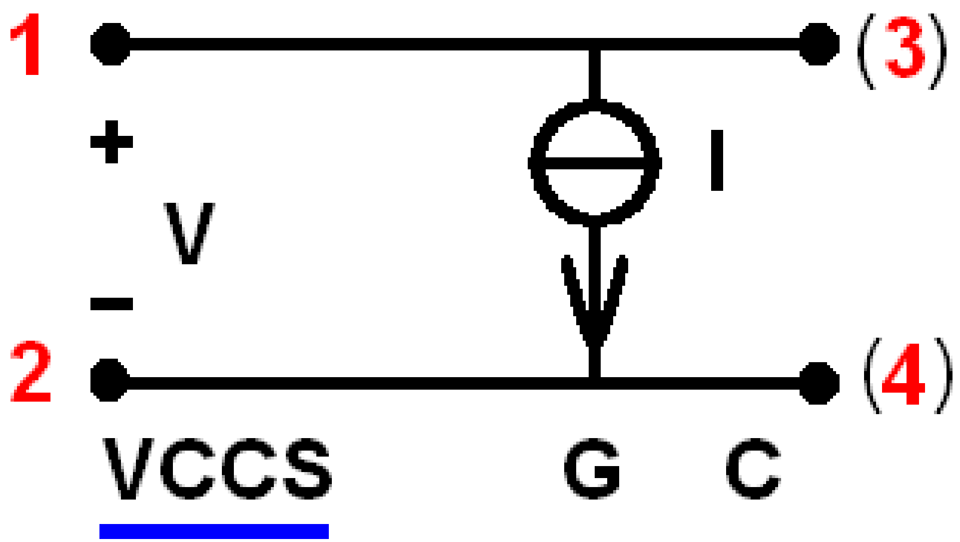
VCVS



CCVS

CSA

ni+ = 1, ni− = 2, no+ = 3, no− = 4

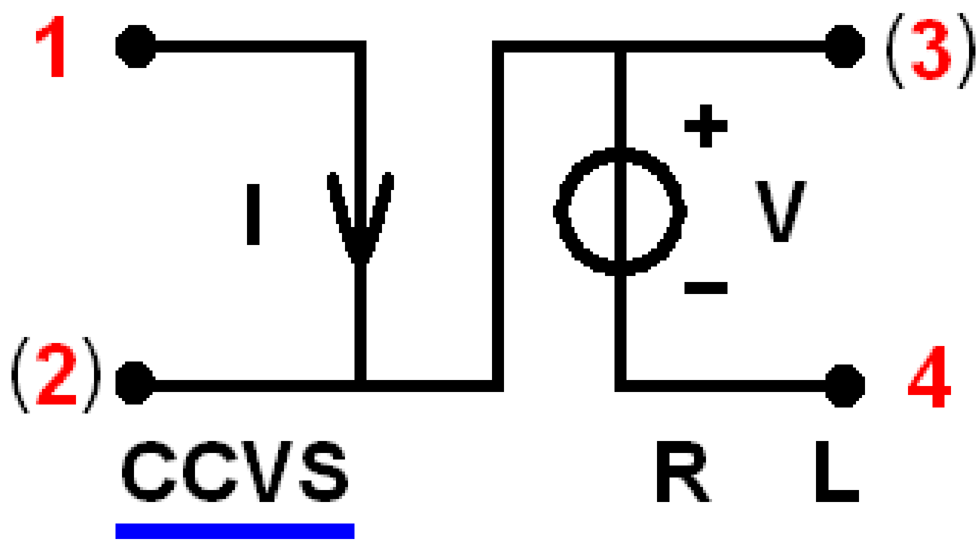


Parallel

Admittance $I = Y(V)$

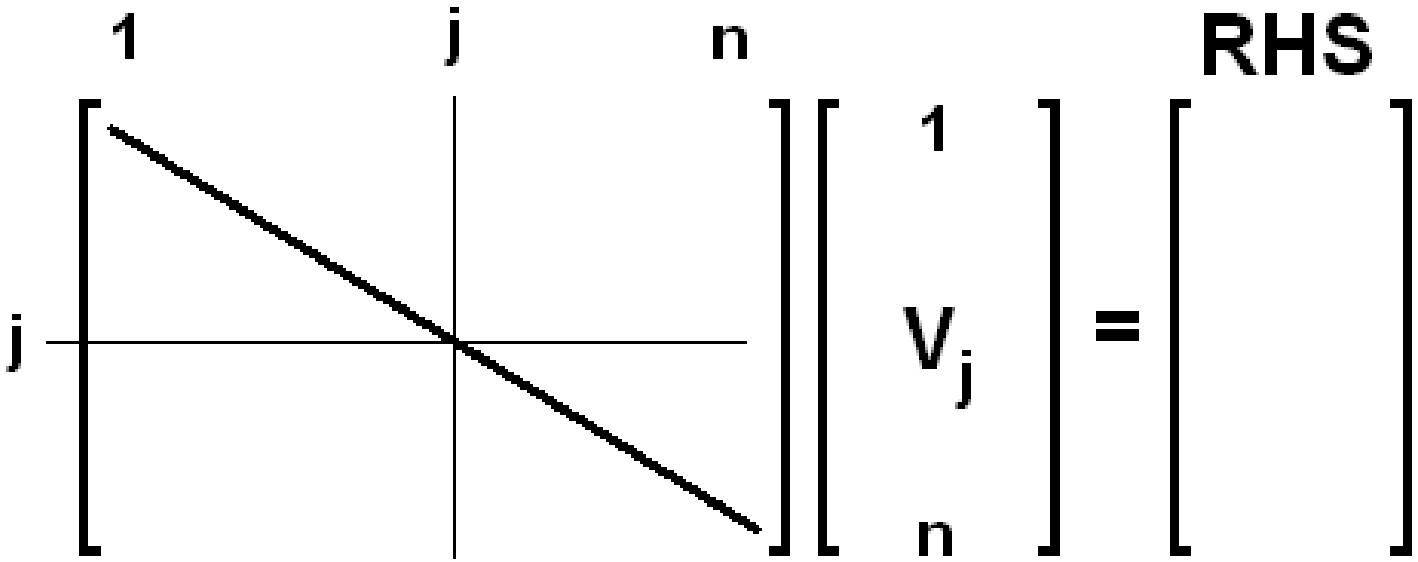
Series

Impedance $V = Z(I)$



Arbitrary Equation System

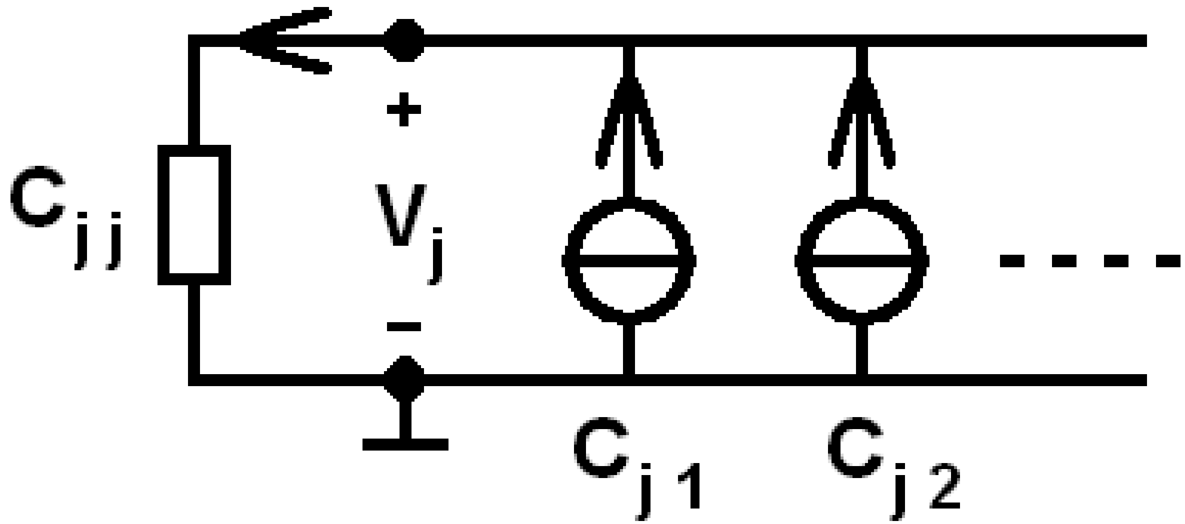
CSA



Rows: Equations **Columns: Variables**

Coefficients:

Controlled Sources



$$V_j = F(V_i)_{i \neq j}$$

Now back to nodal formulation

NODE EQUATIONS

RESTRICTIONS DRAW BACKS

1. NO SHORT-CIRCUITS ($R = 0$, $G = \infty$)
2. NO IDEAL VOLTAGE SOURCES
3. NO IDEAL CURRENT-CONTROLLED SOURCES
4. INTEGRO-DIFFERENTIAL EQUATIONS

→ 2. ORDER DIFFERENTIAL EQUATIONS

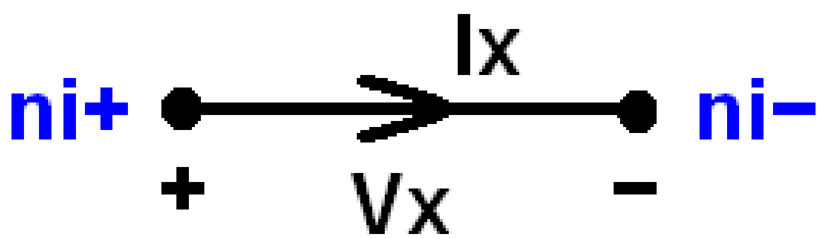
sC , G , $1/(sL)$

R , sL

$\underline{\underline{YN}} * \underline{VN} = - \underline{JS}$

Admittance branches

An admittance branch is a branch for which the branch voltage is the primary variable (the input, the excitation) and the branch current is the secondary variable (the output, the response)



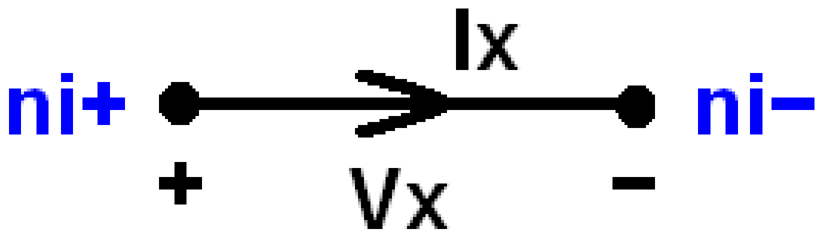
Ix = Y(Vx)

e.g. a capacitor is a current source controlled by the time derivative of the voltage

$$I_C = Y(V_C) = sC * V_C = C * \frac{dV_C}{dt}$$

Impedance branches

An impedance branch is a branch for which the branch current is the primary variable and the branch voltage is the secondary variable



Vx = Z(Ix)

e.g. an inductor is a voltage source controlled by the time derivative of the current

$$V_L = Z(I_L) = sL * I_L = L * \frac{dI_L}{dt}$$

$$\underline{\underline{YN}} * \underline{VN} = -\underline{JS}$$

instead
HYBRID equations

CSA

COLS: VARIABLES

ROWS: EQUATIONS

$$
\begin{array}{c c}
 & \begin{array}{cc} \quad VN \quad & \quad IZ \quad \end{array} \\
\begin{array}{c} KCL \\ \\ OHM \end{array} &
\left[\begin{array}{c|c}
\underline{\underline{Y}} & \underline{\underline{A_Z}} \\
\hline
-\underline{\underline{A_Z^t}} & \underline{\underline{Z}}
\end{array}\right]
\left[\begin{array}{c}
\underline{VN} \\
\hline
\underline{IZ}
\end{array}\right]
=
\left[\begin{array}{c}
-\underline{JS} \\
\hline
-\underline{ES}
\end{array}\right]
\end{array}
$$

CSA

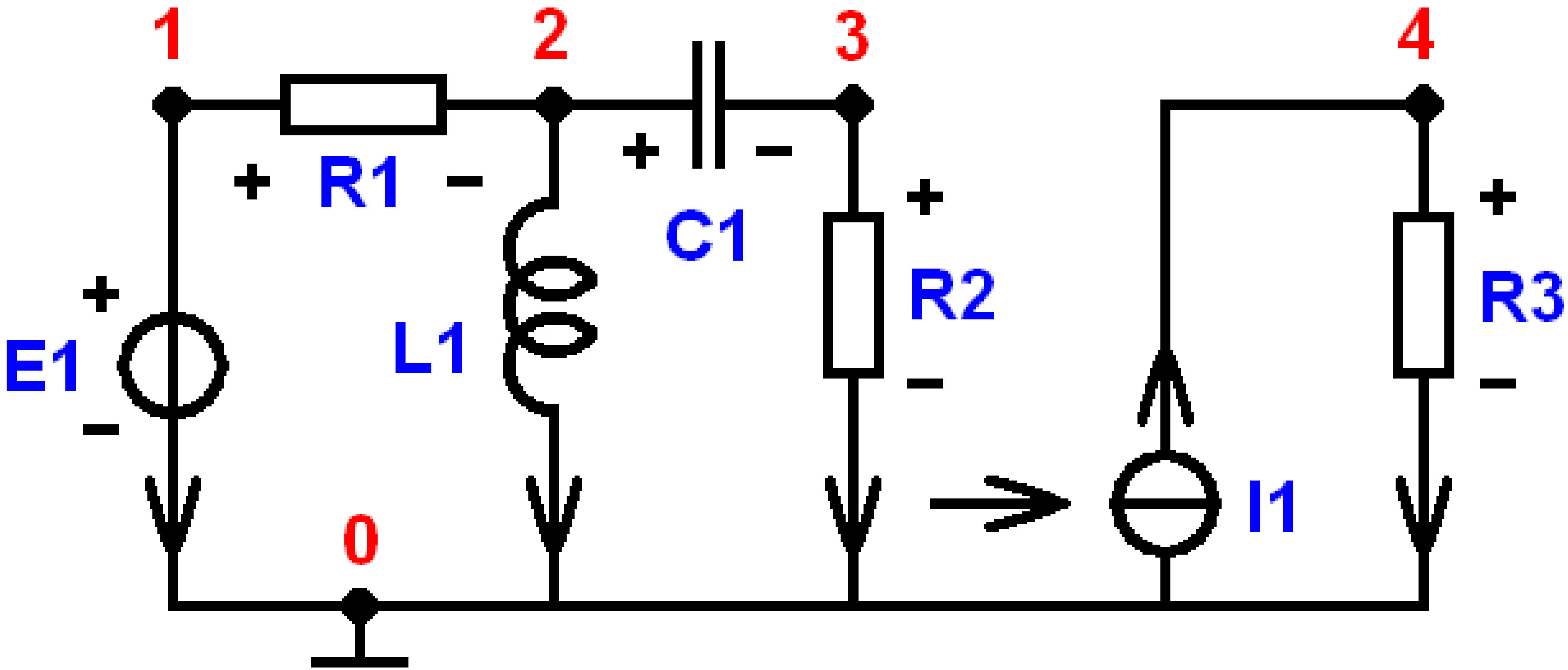
$$
\begin{array}{c}
\begin{array}{cc} \qquad\qquad \text{VN} & \qquad\qquad\qquad\qquad \text{IZ} \end{array} \\
\left[\begin{array}{cc|cc}
\underline{\underline{Y}} & \text{VCCS} & \underline{\underline{A_z}} & \text{CCCS} \\ \hline
-\underline{\underline{A_z^t}} & \text{VCVS} & \underline{\underline{Z}} & \text{CCVS}
\end{array}\right]
\end{array}
\left[\begin{array}{c} \underline{\text{VN}} \\ \hline \underline{\text{IZ}} \end{array}\right]
=
\left[\begin{array}{c} -\underline{\text{JS}} \\ \hline -\underline{\text{ES}} \end{array}\right]
$$

The kernel of the solution of a set of non-linear equations is the iterated solution of a set of linear equations based on Taylor series evaluation

$i = f(v)$, $\quad f(v) = f(0) + v * \text{fdot}(0)$, $\quad \text{fdot} = di/dv$

Dynamic: $g = di/dv$ $\qquad$ Static: $G = I/V$

AN EXAMPLE



E1 = 5 V, R1 = 1 Ω, L1 = 1 H, C1 = 1 F,

R2 = 0 Ω, I1 = 2*IR2, R3 = 3 Ω

NETLIST (LIST OF ELEMENTS)

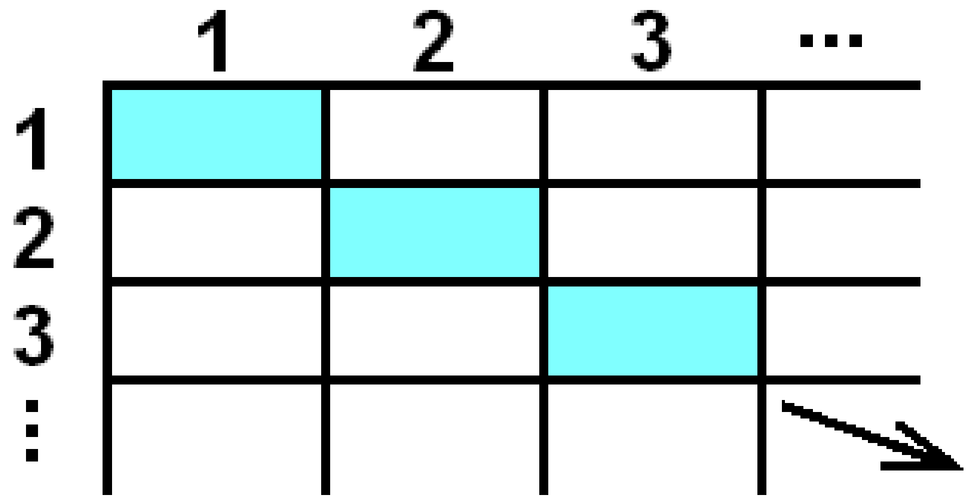
AN EXAMPLE

TYPE-NAME	+NODE	−NODE	VALUE
E1	1	0	5
R1	1	2	1
L1	2	0	1
C1	2	3	1
R2	3	0	0
I1	0	4	2 * I(R2)
R3	4	0	3

- READ ELEMENT
- NEW VARIABLES ?
- NEW EQUATIONS ?

"GROW" THE EQUATIONS

E1	1	0	5



	N1	IE1		RHS
"1"		+1		
"E1"	−1	0		−E1

R1 1 2 1

G1 = 1/R1

	N1	IE1	N2		RHS
"1"	+G1	+1	−G1		
"E1"	−1	0			−E1
"2"	−G1		+G1		

L1 2 0 1

$$-V2 + sL1 * IL1 = 0$$

	N1	IE1	N2	IL1	RHS
"1"	+G1	+1	−G1		
"E1"	−1	0			−E1
"2"	−G1		+G1	+1	
"L1"			−1	sL1	

C1 2 3 1

	N1	IE1	N2	IL1	N3	...	RHS
"1"	+G1	+1	−G1				
"E1"	−1	0					−E1
"2"	−G1		+G1 +sC1	+1	−sC1		
"L1"			−1	sL1			
"3"			−sC1		+sC1		

R2 3 0 0

	N2	IL1	N3	IR2	...	RHS
"2"	+G1 +sC1	+1	−sC1			
"L1"	−1	sL1				
"3"	−sC1		+sC1	+1		
"R2"			−1	0		

I1 0 4 2 * I(R2)

	N2	IL1	N3	IR2	N4	...	RHS
"2"	+G1 +sC1	+1	−sC1				
"L1"	−1	sL1					
"3"	−sC1		+sC1	+1			
"R2"			−1	0			
"4"				−2			

R3 4 0 3

G3 = 1/R3

	N2	IL1	N3	IR2	N4	...	RHS
"2"	+G1 +sC1	+1	−sC1				
"L1"	−1	sL1					
"3"	−sC1		+sC1	+1			
"R2"			−1	0			
"4"				−2	+G3		

	N1	IE1	N2	IL1	N3	IR2	N4
"1"	+G1	+1	−G1				
"E1"	−1	0					
"2"	−G1		+G1 +sC1	+1	−sC1		
"L1"			−1	sL1			
"3"			−sC1		+sC1	+1	
"R2"					−1	0	
"4"						−2	+G3

	N1	N2	N3	N4	IE1	IL1	IR2
"1"	+G1	−G1			+1		
"2"	−G1	+G1 +sC1	−sC1			+1	
"3"		−sC1	+sC1				+1
"4"				+G3			−2
"E1"	−1				0		
"L1"		−1				sL1	
"R2"			−1				0

	IE1	N4	IR2	N2	IL1	N1	N3
"1"	+1			−G1		+G1	
"4"		+G3	−2				
"3"			+1	−sC1			+sC1
"2"				+G1 +sC1	+1	−G1	−sC1
"L1"				−1	sL1		
"E1"	0					−1	
"R2"			0				−1

CONCLUSION

The "Modified Nodal Approach" **MNA**
is compared with a simple strategy for setting up the equations for an electrical or electronic circuit, the "Controlled Source Approach" **CSA**

$$\left[\begin{array}{cc|cc} \underline{\underline{Y}} & \text{VCCS} & \underline{\underline{A_Z}} & \text{CCCS} \\ \hline -\underline{\underline{A_Z}}^t & \text{VCVS} & \underline{\underline{Z}} & \text{CCVS} \end{array}\right] \left[\begin{array}{c} \underline{VN} \\ \hline \underline{IZ} \end{array}\right] = \left[\begin{array}{c} -\underline{JS} \\ \hline -\underline{ES} \end{array}\right]$$

MNA stamps are not necessary

SPICE-programs based on MNA should be reprogrammed

Thank you for your attention